

## Maximum Likelihood Estimation (MLE)

### Concepts

1. We often are not given the actual values for the parameters of different discrete random variables and instead have to estimate what they are. To do this, we calculate the **likelihood function**  $L(\theta)$  which is the probability that we see the data we see if we set the parameter equal to  $\theta$ . Namely,  $L(\theta|x_1, \dots, x_n) = P(x_1, \dots, x_n|\theta)$ , the probability we see  $x_1, \dots, x_n$  if our parameter is equal to  $\theta$ . Then we choose the value of  $\theta$  that maximizes this function by taking the derivative and setting it equal to 0.

### Examples

2. Let  $f(x) = \frac{1}{2\sqrt{2\pi}}e^{-(x-5)^2/8}$  be a PDF. Calculate the probability  $P(3 \leq X \leq 7)$ .
3. A Pareto distribution is given by the PDF  $f(x) = \frac{p}{x^{p+1}}$  for  $x \geq 1$  and 0 for  $x < 1$  for some parameter  $p$ . Suppose I draw from this distribution 4 times and get the values 1, 2, 1, 1, 1. What is the 95% confidence interval for  $\mu$ ?
4. I have a bag with 5 red and blue balls. I pull out a ball and it is red. I put it back and I add 3 blue balls and pull out another ball, which is blue. What is the maximum likelihood for the original number of blue balls.
5. You assume that the lifespan of lightbulbs are exponentially distributed (PDF is  $\lambda e^{-\lambda t}$  for  $t \geq 0$ ) and notice that your three light bulbs go out in 1, 2, and 3 years. What is the maximum likelihood estimator for  $\lambda$ ?

### Problems

6. True    False    In MLE, you calculate the probability that your parameter  $\theta$  is a given value.
7. True    False    In the ball example from above, the solutions is always found by setting the derivative to 0.
8. True    False    The likelihood function to determine the probability of flipping heads with a biased coin if we see 3 heads out of 5 flips is a polynomial of degree 5.

9. The number of students in a class that can Dougie is normally distributed mean 15 and standard deviation 4. Calculate the probability that in this class between 16 and 20 students can Dougie.
10. The number of students in a class that knows how to do “The Woah” is normally distributed with mean 5 and standard deviation 2. Calculate the probability that between 4 and 8 students know how to do it.
11. There is a bag with 12 balls colored red and blue. You pull out three balls (with replacement) and get *BBR*. What is the maximum likelihood for the number of blue balls in bag?
12. I go to Kip’s and want to figure out the total number of students  $n$  there. By looking, I see that I’ve taught 2 of the students there. I pick a student at random and it turns out to be one of the students I’ve taught. Then 8 more students come in and of those I have taught 7 of them. Now I again pick a random student and I haven’t taught this second student picked. What is the most likely number  $n$  of total students at Kip’s originally?
13. You have a coin that you think is biased. you flip it 4 times and get the sequence *HHHT*. What is the maximum likelihood estimate for the probability of getting heads?
14. During Cal Day, my two friends and I asked prospective students where they were from until we found someone who wasn’t from California. We had to ask 23, 18, 46 people respectively before finding someone not from California. What is the maximum likelihood estimate for the percentage of students from California?